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Kohji Yamamura · Hajime Katsumata

Estimation of the probability of insect pest introduction through imported commodities

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Abstract A quantitative risk assessment is needed for each quarantine pest insect to ensure quarantine security without sacrificing the transparency of international trade. The probability of introduction, which is defined as the probability that one or more reproductive individuals of a pest insect species pass the port, is one of the basic components determining the risk of pest invasion. The probability depends on two biological characteristics of pests: mode of reproduction and spatial distribution of insects per host plant. In this article, the probability of introduction was calculated for each of the following four categories: (1) bisexual, gregarious pests; (2) bisexual, solitary pests; (3) parthenogenetic, gregarious pests; and (4) parthenogenetic, solitary pests. Then, equations were derived to predict the effects of two prevention practices conducted before export: disinfestation treatment and the subsequent export sampling inspection of consignments. These equations also enable estimation of the probability of introduction under natural mortality, which thus can be used in place of the criterion of Maximum Pest Limit (MPL). The method was applied to the Mexican fruit fly *Anastrepha ludens* (Loew), as an example. The contour graph of the probability of introduction indicated the optimal combination of the intensity of two prevention practices that ensures a given security level. Existence of an antagonistic interaction was also indicated between the disinfestation treatment and the subsequent sampling inspection.

Key words Quarantine treatments · Sampling inspection · Invasion · Mexican fruit fly

K. Yamamura (✉)
Laboratory of Population Ecology, National Institute of Agro-Environmental Sciences, 3-1-1 Kannondai, Tsukuba 305-8604, Japan
Tel. +81-298-38-8313; Fax +81-298-38-8199
e-mail: yamamura@niaes.affrc.go.jp

H. Katsumata
Yokohama Plant Protection Station, Yokohama, Japan

Introduction

The number of invading species transported by means of imported commodities is exponentially increasing in Japan (Kiritani 1998). Recently, it was confirmed that the Mexican bean beetle, *Epilachna varivestis* Mulsant, has invaded Japan, causing considerable damage in *Phaseolus* fields in central Honshu (Fujiyama et al. 1998). To prevent such an invasion without sacrificing the transparency of international trade, a quantitative risk assessment is needed for each quarantine pest. Successful invasion by a pest species is related to many factors, such as the climatic conditions at the destination, the existence of natural enemies, and the possibility of pest arrival. The risk assessment must consider these various components. In this article, we focus on one of these components: the probability that one or more reproductive individuals pass the port. We call it the probability of introduction, for simplicity. A successful introduction of a pest, by definition, does not always result in the establishment of the species because other conditions, such as climatic factors, may not be suitable.

The probability of successful introduction depends on two biological characteristics of pests: the mode of reproduction and the spatial distribution of insects per host plant. Bisexually reproducing pests are generally less liable to be introduced compared to parthenogenetic pests because they cannot yield reproductive individuals if their density is too low to find mates. If the spatial distribution of adults is aggregated, however, bisexual pests also reproduce as easily as parthenogenetic pests as they can find mates easily. The spatial distribution of adults is greatly influenced by the distribution of larvae per plant. Larvae of some pests such as the Mexican fruit fly live gregariously in an infested fruit. We call such pests gregarious pests. Larvae of other pests such as codling moths mostly live singly in an infested fruit. We call such pests solitary pests. The distribution of emerging adults of gregarious pests is more aggregated than that of solitary pests, and hence gregarious pests have a larger probability of introduction. Therefore, it is preferable to discuss the probability of introduction for each of the following four categories: (1) bisexual, gregarious pests; (2) bisexual, solitary pests; (3) parthenogenetic, gregarious pests; and (4) parthenogenetic, solitary pests. Landolt et al. (1984) and Vail et al. (1993) derived an equation predicting the probability of introduction for bisexual, solitary pests. However, they calculated only the probability of introduction for each consignment. Many consignments are actually involved in each trade, and hence the probability must be evaluated so as to include all consignments passing the port.

Two quarantine practices are now frequently used to prevent the invasion of pests: one is disinfestation treatment before export and the other is the inspection of consignments by sampling. These practices require considerable expense, and hence we should quantitatively estimate their effectiveness to determine the optimal intensity of these practices. In this article, we first calculate the probability of introduction of fruit pests for each of the four categories. Then, we derive equations to predict the effects of prevention practices. These equations are applied to the data of the Mexican fruit fly, *Anastrepha ludens* (Loew), from Mangan et al. (1997).

Model structure

Assumptions

1. The proportion of fruits that contain one or more live larvae (infested fruits) varies depending on the production area and the year. A gamma distribution can approximately describe the probability distribution of the proportion of infested fruits in the production area of a given consignment.
2. The number of live larvae in an infested fruit is approximately described by a logarithmic distribution that is independent of the proportion of infested fruits.
3. The sex ratio is constant.
4. Every consignment contains fruits that were drawn at random from the infinite population of the production area.
5. An disinfestation treatment is conducted before export. Each larva is killed by a constant probability.
6. A sampling inspection is conducted after the disinfestation treatment. A sample is drawn at random from every consignment.
7. If the sample contains one or more live larvae, the consignment is discarded. Otherwise, the consignment is shipped.
8. Larvae in consignments emerge as adults during the shipping.
9. An emerged female always mates successfully if one or more male adults exist in the same consignment.

Notation

k = the number of consignments imported during a given period

n_i = the number of fruits in the ith consignment ($i = 1,2,\ldots,k$)

W_i = the proportion of infested fruits in the production area of the ith consignment ($i = 1,2,\ldots,k$)

σ^2 = the variance of W_i

V_i = the number of infested fruits in the ith consignment ($i = 1,2,\ldots,k$)

Z_i = the number of live larvae in the ith consignment ($i = 1,2,\ldots,k$)

Y_i = the number of females in the ith consignment ($i = 1,2,\ldots,k$)

H_i = the number of reproductive females in the ith consignment ($i = 1,2,\ldots,k$); for parthenogenetic pests, H_i equals Z_i; for bisexual pests, H_i equals the number of mated females

f = the probability that an individual is a female

X = the number of live larvae in an infested fruit

p = the probability of survival after the disinfestation treatment

C_i = the number of live larvae in the ith consignment after the disinfestation treatment ($i = 1,2,\ldots,k$)

s_i = the number of fruits sampled from the ith consignment after the disinfestation treatment ($i = 1,2,\ldots,k$)

Q_i = the number of live larvae in the sample drawn from the ith consignment after the disinfestation treatment ($i = 1,2,\ldots,k$)

R = the probability that one or more reproductive insects pass the port during a given period

k' = the number of consignments from which a sample is drawn before the disinfestation treatment for the estimation of parameters

s'_i = the number of fruits sampled from the ith consignment before the disinfestation treatment for the estimation of parameters ($i = 1,2,\ldots,k'$)

Q'_i = the number of live larvae in the sample drawn from the ith consignment before the disinfestation treatment for the estimation of parameters ($i = 1,2,\ldots,k'$)

U'_i = the number of infested fruits in the sample drawn from the ith consignment before the disinfestation treatment for the estimation of parameters ($i = 1,2,\ldots,k'$)

Distribution

The probability distribution of the proportion of infested fruits is

$$g(w) = \frac{1}{\Gamma(a)} b^a w^{a-1} \exp(-bw) \qquad (0 \le w) \tag{1}$$

where a is a shape parameter, b is a scale parameter, and $\Gamma(\cdot)$ is the gamma function. The mean and variance are given by a/b and a/b^2, respectively. A beta distribution is generally most suitable for the description of the probability defined in the interval (0,1), because it has sufficient flexibility in describing various forms of distribution. As discussed by Yamamura and Sugimoto (1995), a beta distribution is approximately described by a gamma distribution if the proportion of infested fruits is small. For this reason, we use the gamma distribution throughout this article.

The distribution of the number of live larvae in an infested fruit (X) is determined mainly by the distribution of the number of eggs laid on a fruit by a single female, unless the density of females is high enough to yield multiple oviposition. Hence, the assumption that X is independent of W_i seems to be reasonable. The distribution of X is given by

$$\Pr(X = x) = \frac{\alpha\beta^x}{x} \qquad (x = 1, 2, \ldots) \tag{2}$$

where $\alpha = -1/\ln(1-\beta)$. The mean and variance are given by $\alpha\beta/(1-\beta)$ and $\alpha\beta(1-\alpha\beta)/(1-\beta)^2$, respectively. At the limit, $\beta \to 0$, the quantity of Eq. 2 becomes 1 for $x = 1$ and 0 for $x > 1$, which corresponds to the case of solitary pests by definition. Thus, we can describe both gregarious pests and solitary pests by changing the parameter β in Eq. 2. Furthermore, Eq. 2 is mathematically tractable as discussed below. Hence, we use Eq. 2 throughout for convenience. Actual distribution of the number of live larvae per infested fruit may be slightly different from Eq. 2. If females lay egg masses of nearly the same size on each fruit, for example, the variance of X may become smaller than that of the logarithmic distribution. In such a case, the degree of aggregation of adults will be overestimated and the probability of introduction will be overestimated accordingly.

The distribution of the number of females for a given number of larvae is given by the binomial distribution:

$$\Pr(Y_i = y \mid Z_i = z) = \binom{z}{y} f^y (1-f)^{z-y} \qquad (y = 0, 1, 2, \ldots, z) \tag{3}$$

The distribution of the number of infested fruits in the ith consignment of $W_i = w$ is given by the Poisson distribution if w is small:

$$\Pr(V_i = v \mid W_i = w) = \frac{(wn_i)^v \exp(-wn_i)}{v!} \qquad (v = 0, 1, 2, \ldots) \tag{4}$$

From Eqs. 2 and 4, the distribution of the number of live larvae in the ith consignment is given by a negative binomial distribution (Quenouille 1949):

$$\Pr(Z_i = z \mid W_i = w) = \binom{-\alpha w n_i}{z} (-\beta)^z (1-\beta)^{\alpha w n_i} \qquad (z = 0, 1, 2, \ldots) \tag{5}$$

The mean and variance are given by $\alpha\beta wn_i/(1-\beta)$ and $\alpha\beta wn_i/(1-\beta)^2$, respectively. For solitary pests, Eq. 5 becomes a simple form as the limit $\beta \to 0$:

$$\Pr(Z_i = z \mid W_i = w) = \frac{(wn_i)^z \exp(-wn_i)}{z!} \qquad (z = 0, 1, 2, \ldots) \tag{6}$$

which is the same form as Eq. 4 as expected intuitively.

The probability that no consignment contains reproductive females is given by

$$\prod_{i=1}^{k} \left[1 - \Pr(H_i \geq 1)\right]$$

Hence, the probability of introduction is given by subtracting the above probability from 1:

$$R = 1 - \prod_{i=1}^{k} \left[1 - \Pr(H_i \geq 1)\right] \tag{7}$$

If $\Pr(H_i \geq 1)$ is sufficiently small, we obtain the following approximation by expanding the right-hand side of Eq. 7:

$$R \approx \sum_{i=1}^{k} \Pr(H_i \geq 1) \tag{8}$$

Probability of introduction

Bisexual, gregarious pests

If at least two individuals exist in a consignment, at least one mated female is produced except for the following two cases: all the individuals are females or all the individuals are males. Hence, the probability that at least one mated female exists in a consignment of $W_i = w$ is given by subtracting these two probabilities from 1 (see Appendix for derivation):

$$\begin{aligned} &\Pr(H_i \geq 1 \mid W_i = w) \\ &= \sum_{z=1}^{\infty} \left[1 - \Pr(Y_i = z \mid Z_i = z) - \Pr(Y_i = 0 \mid Z_i = z)\right] \Pr(Z_i = z \mid W_i = w) \\ &= 1 - \left(\frac{1-\beta}{1-f\beta}\right)^{\alpha w n_i} - \left(\frac{1-\beta}{1-(1-f)\beta}\right)^{\alpha w n_i} + (1-\beta)^{\alpha w n_i} \end{aligned} \tag{9}$$

The probability that at least one mated female exists in a given consignment is obtained by integrating w out in Eq. 9 (see Appendix for derivation):

$$\begin{aligned} &\Pr(H_i \geq 1) \\ &= \int_0^{\infty} \Pr(H_i \geq 1 \mid W_i = w) g(w) dw \\ &= 1 - \left\{1 - \frac{\alpha n_i}{b}\ln(1-\beta) + \frac{\alpha n_i}{b}\ln(1-f\beta)\right\}^{-a} \\ &\quad - \left\{1 - \frac{\alpha n_i}{b}\ln(1-\beta) + \frac{\alpha n_i}{b}\ln\left[1-(1-f)\beta\right]\right\}^{-a} \\ &\quad + \left\{1 - \frac{\alpha n_i}{b}\ln(1-\beta)\right\}^{-a} \end{aligned} \tag{10}$$

When the variance of $g(w)$ approaches zero, Eq. 10 coincides with Eq. 9 with w replaced by a/b. Although we assumed that all the larvae emerge as adults during the shipping, some larvae may emerge after arriving at the port. If they emerge after being sparsely distributed toward the market, the opportunity of mating decreases, and hence Eq. 10 will overestimate the probability of introduction in such a case.

Bisexual, solitary pests

We obtain the following equations by considering the limit, $\beta \to 0$, of Eqs. 9 and 10:

$$\begin{aligned}\Pr\left(H_i \geq 1 \middle| W_i = w\right) &= 1 - \exp\left[-wn_i(1-f)\right] - \exp(-wn_i f) + \exp(-wn_i) \\ &= \left\{1 - \exp\left[-wn_i(1-f)\right]\right\}\left\{1 - \exp(-wn_i f)\right\}\end{aligned} \tag{11}$$

$$\begin{aligned}\Pr\left(H_i \geq 1\right) &= 1 - \left(1 + \frac{n_i(1-f)}{b}\right)^{-a} - \left(1 + \frac{n_i f}{b}\right)^{-a} + \left(1 + \frac{n_i}{b}\right)^{-a}\end{aligned} \tag{12}$$

Equation 11 is the formula derived by Landolt et al. (1984) and Vail et al. (1993).

Parthenogenetic pests

In this case, we have $\Pr(H_i \geq 1|W_i = w) = \Pr(Z_i \geq 1|W_i = w) = \Pr(V_i \geq 1|W_i = w)$. Hence, we simply obtain:

$$\begin{aligned}\Pr\left(H_i \geq 1\right) &= \int_0^\infty \Pr\left(Z_i \geq 1 \middle| W_i = w\right) g(w)\,dw \\ &= 1 - \left(1 + \frac{n_i}{b}\right)^{-a}\end{aligned} \tag{13}$$

Effects of prevention practices

Let us consider a case in which the same disinfestation treatment is performed for all consignments. The probability that c individuals survive among z individuals is given by a binomial distribution:

$$\binom{z}{c} p^c (1-p)^{z-c} \tag{14}$$

Hence, the distribution of the number of individuals that survived after the disinfestation treatment in a consignment of $W_j = w$ is given by a negative binomial distribution (see Appendix for derivation):

$$\begin{aligned}\Pr\left(C_i = c \middle| W_i = w\right) &= \sum_{z=c}^{\infty} \binom{z}{c} p^c (1-p)^{z-c} \Pr\left(Z_i = z \middle| W_i = w\right) \\ &= \binom{-\alpha w n_i}{c} \left(\frac{-\beta p}{1-\beta(1-p)}\right)^c \left(1 - \frac{\beta p}{1-\beta(1-p)}\right)^{\alpha w n_i}\end{aligned} \tag{15}$$

We can obtain the equation for solitary pests by considering the limit, $\beta \to 0$, of Eq. 15:

$$\Pr\left(C_i = c \middle| W_i = w\right) = \frac{(wpn_i)^c \exp(-wpn_i)}{c!} \tag{16}$$

We next consider the effect of sampling inspection conducted after the disinfestation treatment. The sample drawn from a consignment derived from a population of $W_i = w$ is identical to a random sample drawn from a population of $W_i = w$. Hence, we obtain the following equation by replacing n_i in Eq. 15 by s_j:

$$\begin{aligned}\Pr\left(Q_i = q \middle| W_i = w\right) &= \binom{-\alpha w s_i}{q} \left(\frac{-\beta p}{1-\beta(1-p)}\right)^q \left(1 - \frac{\beta p}{1-\beta(1-p)}\right)^{\alpha w s_i}\end{aligned} \tag{17}$$

The consignment is accepted when $Q_i = 0$, and hence the probability of acceptance is given by

$$\Pr\left(Q_i = 0 \middle| W_i = w\right) = \left(1 - \frac{\beta p}{1-\beta(1-p)}\right)^{\alpha w s_i} \tag{18}$$

Pest individuals pass the port through the unexamined $(n_i - s_i)$ fruits only if there is no live larva in the sample. We obtain the probability that the unexamined $(n_i - s_i)$ fruits contain z live larvae by replacing n_i in Eq. 15 by $(n_i - s_i)$:

$$\binom{-\alpha w(n_i - s_i)}{z} \left(\frac{-\beta p}{1-\beta(1-p)}\right)^z \cdot \left(1 - \frac{\beta p}{1-\beta(1-p)}\right)^{\alpha w(n_i - s_i)} \tag{19}$$

Hence, we obtain the distribution of the number of larvae by multiplying Eq. 18 and quantity 19:

$$\Pr\left(Z_i = z \middle| W_i = w\right) = \begin{cases} 1 - \left(1 - \dfrac{\beta p}{1-\beta(1-p)}\right)^{\alpha w s_i} + \left(1 - \dfrac{\beta p}{1-\beta(1-p)}\right)^{\alpha w n_i} & (z = 0) \\ \dbinom{-\alpha w(n_i - s_i)}{z} \left(\dfrac{-\beta p}{1-\beta(1-p)}\right)^z \cdot \left(1 - \dfrac{\beta p}{1-\beta(1-p)}\right)^{\alpha w n_i} & (z = 1, 2, \ldots) \end{cases} \tag{20}$$

Using Eq. 20, we can derive the probability of introduction by a method similar to that used in the derivation of Eqs. 10, 12, and 13.

Bisexual, gregarious pests

$$\begin{aligned}\Pr(H_i \geq 1) = & \left(1 - \frac{\alpha n_i}{b}\ln\left(\frac{1-\beta}{1-(1-p)\beta}\right)\right)^{-a} \\ & + \left(1 - \frac{\alpha s_i}{b}\ln\left(\frac{1-\beta}{1-(1-p)\beta}\right)\right)^{-a} \\ & - \left(1 - \frac{\alpha n_i}{b}\ln\left(\frac{1-\beta}{1-(1-p)\beta}\right)\right. \\ & \left. + \frac{\alpha(n_i - s_i)}{b}\ln\left(\frac{1-(1-p(1-f))\beta}{1-(1-p)\beta}\right)\right)^{-a} \\ & - \left(1 - \frac{\alpha n_i}{b}\ln\left(\frac{1-\beta}{1-(1-p)\beta}\right)\right. \\ & \left. + \frac{\alpha(n_i - s_i)}{b}\ln\left(\frac{1-(1-pf)\beta}{1-(1-p)\beta}\right)\right)^{-a}\end{aligned} \tag{21}$$

Equation 21 reduces to Eq. 10 when $s_i = 0$ and $p = 1$. The equation derived by Landolt et al. (1984) and Vail et al. (1993), i.e., Eq. 11, corresponds to a special case of Eq. 21 in which $s_i = 0$, $p = 1$, $\beta \to 0$, and $b \to \infty$.

Bisexual, solitary pests

$$\begin{aligned}\Pr(H_i \geq 1) = & \left(1 + \frac{pn_i}{b}\right)^{-a} + \left(1 + \frac{ps_i}{b}\right)^{-a} \\ & - \left(1 + \frac{p(fn_i + s_i - fs_i)}{b}\right)^{-a} \\ & - \left(1 + \frac{p(n_i - fn_i + fs_i)}{b}\right)^{-a}\end{aligned} \tag{22}$$

Parthenogenetic, gregarious pests

$$\begin{aligned}\Pr(H_i \geq 1) = & \left(1 - \frac{\alpha s_i}{b}\ln\left(\frac{1-\beta}{1-(1-p)\beta}\right)\right)^{-a} \\ & - \left(1 - \frac{\alpha n_i}{b}\ln\left(\frac{1-\beta}{1-(1-p)\beta}\right)\right)^{-a}\end{aligned} \tag{23}$$

Parthenogenetic, solitary pests

$$\Pr(H_i \geq 1) = \left(1 + \frac{ps_i}{b}\right)^{-a} - \left(1 + \frac{pn_i}{b}\right)^{-a} \tag{24}$$

Estimation of parameters

We can estimate the parameters of Eq. 1 by drawing samples at random from consignments that were selected at random before the disinfestation treatment. The distribution of U'_i for $W_i = w$ is expressed by a Poisson distribution if w is small:

$$\Pr\left(U'_i = u \middle| W_i = w\right) = \frac{(ws'_i)^u \exp(-ws'_i)}{u!} \quad (u = 0, 1, 2, \ldots) \tag{25}$$

From Eqs. 1 and 25, the distribution of the number of infested fruits in a given sample is given by a negative binomial distribution (Yamamura and Sugimoto 1995):

$$\begin{aligned}\Pr(U'_i = u) &= \int_0^\infty \Pr\left(U'_i = u \middle| W_i = w\right) g(w)\,dw \\ &= \binom{-a}{u}\left(1 + \frac{s'_i}{b}\right)^{-a}\left(-\frac{s'_i}{b + s'_i}\right)^u \\ & \qquad (u = 0, 1, 2, \ldots)\end{aligned} \tag{26}$$

The mean and variance are given by $s'_i a/b$ and $s'_i a(b + s'_i)/b^2$, respectively. The maximum likelihood estimates of parameters a and b are obtained using the method given by the Appendix of Yamamura and Sugimoto (1995). We can obtain the maximum likelihood estimate of β, which is denoted by $\hat{\beta}$, by iteratively finding the $\hat{\beta}$ satisfying the following equation, considering the conditional likelihood of Q'_i for given U'_i (see Appendix for derivation):

$$\sum_{i=1}^{k'} Q'_i \Big/ \sum_{i=1}^{k'} U'_i = -\frac{\hat{\beta}}{(1-\hat{\beta})\ln(1-\hat{\beta})} \tag{27}$$

Example

Invasion by the Mexican fruit fly has been of concern in the citrus industry in the United States. Mangan et al. (1997) collected ecological parameters including the proportion of fruit infested and the number of pests per infested fruit under various pest management scenarios for mangoes and citrus in regions of Mexico that are infested with the Mexican fruit fly. We applied the foregoing equation to these data except for the biased data of the ground sample of grapefruit. To estimate the distribution of the proportion of infested fruits in the export area, we should draw samples at random from consignments that were selected at random before the disinfestation treatment. For convenience, we

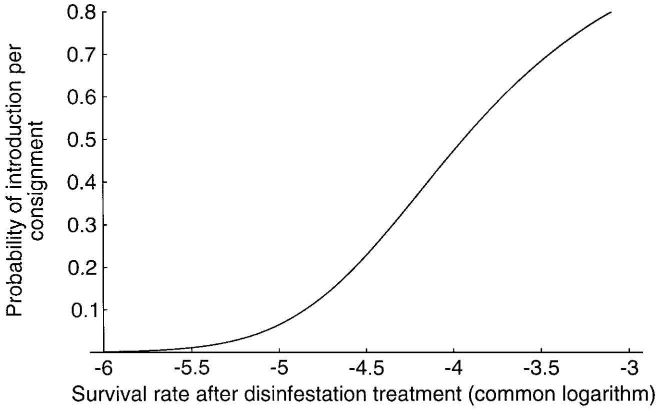

Fig. 1. Reduction of the probability of introduction per consignment by the disinfestation treatment of the Mexican fruit fly. Sampling inspection is not conducted ($s_i = 0$)

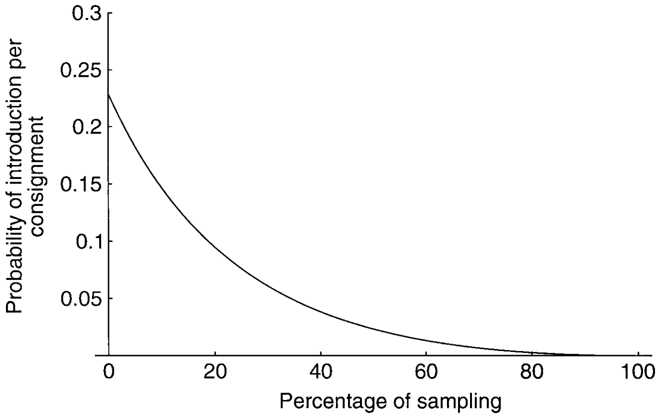

Fig. 2. Reduction of the probability of introduction per consignment by the sampling inspection of the Mexican fruit fly. Disinfestation treatment of Probit 9 mortality is conducted ($p = 32 \times 10^{-6} = 10^{-4.5}$)

treated the data of Mangan et al. (1997) as such a sample, and obtained the maximum likelihood estimates $\hat{a} = 0.523$ and $\hat{b} = 5.42$. The total number of infested fruits and the total number of pupae observed were $\Sigma_{i=1}^{k'} U_i' = 2564$ and $\Sigma_{i=1}^{k'} Q_i' = 16419$, respectively. Then, we obtained the maximum likelihood estimate $\hat{\beta} = 0.924$ from Eq. 27. We used $f = 0.5$ and $n_i = 100000$ for all the consignments, for simplicity.

Figures 1 and 2 indicate the effect of disinfestation treatment and the sampling inspection on the probability of introduction per consignment calculated by Eq. 21, respectively. The contour in Fig. 3 expresses the probability as the function of the intensity of two quarantine practices. If the size of consignment is constant, we can obtain the total probability of introduction by multiplying the contour value by the total number of consignments imported, as indicated by Eq. 8. Therefore, if we want to keep the probability of introduction below 10^{-3} when the total number of consignments is 10^4, for example, we must keep the probability of introduction per consignments below $10^{-3}/10^4 = 10^{-7}$. We can obtain the combination of the disinfestation treatment

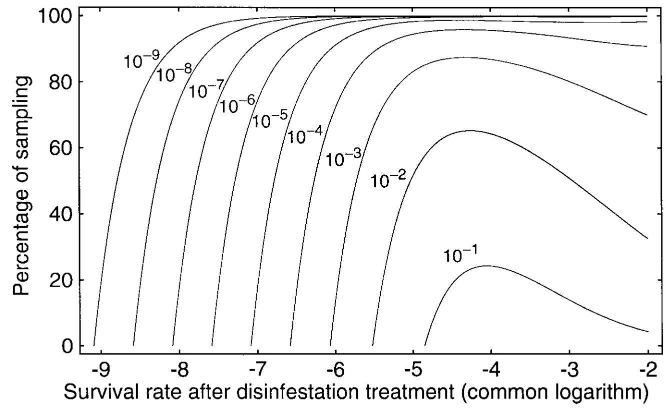

Fig. 3. Contour of the probability of introduction per consignment for the Mexican fruit fly expressed as the function of the intensity of sampling inspection and disinfestation treatment. The contour readily indicates the combination of the percentage of sampling and the disinfestation treatment to achieve a given probability of introduction

and sampling inspection to achieve this probability by using the contour of 10^{-7} in Fig. 3.

Discussion

We have provided a method to evaluate the effects of quarantine practices on the reduction of the probability of introduction. For simplicity, we assumed that all larvae emerge to adults successfully in the consignment during the shipment, although actual larvae and pupae are subject to natural mortality. When there is no sampling inspection, natural mortality has an effect similar to that of the disinfestation mortality. In this case, therefore, we can estimate the probability of introduction under natural mortality by using the same equations. For example, if the natural mortality is 0.5, we can estimate the probability by substituting $p = 0.5$ and $s_i = 0$ into Eq. 21. Baker et al. (1990) proposed the concept of maximum pest limit (MPL) to solve the same problem. Their argument is as follows. Larvae of the Queensland fruit fly *Bactrocera tryoni* (Froggatt) have to survive at least 2–3 weeks before emerging as adults. The natural mortality is 20% per week. When an infested fruit with three live larvae is kept in a suitable site, therefore, natural mortality factors should ensure that fewer than two individuals emerge after 3 weeks. Hence, they considered that "a population of three live immature fruit flies entering New Zealand will not result in a mating pair (i.e., a single introduction at this level of infestation does not pose any danger to New Zealand's quarantine security)." Based on this argument, which is not exactly correct, they assumed that MPL = 3. The concept of MPL, therefore, will not be necessary if we can directly estimate the probability of yielding a mating pair under the natural mortality using Eqs. 21, 22, 23, or 24.

We considered two kinds of prevention practices conducted before export: disinfestation treatment and subsequent sampling inspection. Various protection practices are actually used in the quarantine procedure. Recently, a quarantine procedure called the systems approach has been proposed (Hata et al. 1992; Jang and Moffitt 1994; Jang 1996). This approach aims at achieving total quarantine security by integrating the protection efforts conducted in several production phases including integrated pest management in field and inspection of packed fruits. If the mortality in each phase is mutually independent, we can estimate the total survival rate by multiplying the survival rate of each phase. If there is some interaction between the mortality, however, the estimation becomes complicated.- Mangan and Sharp (1994) reexamined the experimental results of multiple treatment from the literature to clarify the existence of interactions. The data of von Windeguth and Gould (1990) indicated no consistent interaction between the effects of gamma radiation and cold storage. However, synergetic effects were detected between the methylbromide treatment and cold storage reported by Seo et al. (1971) and between the hot water dip and cold storage reported by Couey et al. (1984). In contrast, Fig. 3 indicates the existence of antagonistic effects between the disinfestation treatment and the subsequent sampling inspection. When the survival rate under the disinfestation treatment is large, such as $p = 10^{-2}$, the slope of the contour is steep along the vertical direction, indicating that the probability of introduction considerably decreases with increasing percentage of sampling. When the survival rate under the disinfestation treatment is small, such as $p = 10^{-8}$, the slope is gentle along the vertical direction, indicating that the efficiency of sampling inspection is small. Thus, the efficiency of sampling inspection depends on the intensity of disinfestation treatment. Yamamura and Katsumata (1999) provided formulae to estimate the average proportion of infested fruits, and showed the existence of a similar antagonistic interaction between quarantine treatment and sampling inspection. Generally, the combined effects of multiple treatments should be estimated carefully for each combination of treatments.

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Appendix

Derivation of Eq. 9

We obtain the following equation using Eqs. 3 and 5:

$$\begin{aligned}
&\Pr\left(H_i \geq 1 \middle| W_i = w\right) \\
&\quad = \sum_{z=1}^{\infty}\Big[1 - \Pr\left(Y_i = z \middle| Z_i = z\right) \\
&\qquad - \Pr\left(Y_i = 0 \middle| Z_i = z\right)\Big]\Pr\left(Z_i = z \middle| W_i = w\right) \\
&\quad = \sum_{z=1}^{\infty}\left(1 - f^z - (1-f)^z\right)\binom{-\alpha w n_i}{z}(-\beta)^z (1-\beta)^{\alpha w n_i}
\end{aligned} \tag{A1}$$

Here, we can use the binomial theorem for any real number u and a real number v satisfying $-1 < v < 1$:

$$\sum_{i=0}^{\infty}\binom{u}{i} v^i = (1+v)^u \tag{A2}$$

Hence, we obtain:

$$\begin{aligned}
&\Pr\left(H_i \geq 1 \middle| W_i = w\right) \\
&\quad = \Big\{\left[(1-\beta)^{-\alpha w n_i} - 1\right] - \left[(1-f\beta)^{-\alpha w n_i} - 1\right] \\
&\qquad - \left[\left(1-(1-f)\beta\right)^{-\alpha w n_i} - 1\right]\Big\}(1-\beta)^{\alpha w n_i} \\
&\quad = 1 - \left(\frac{1-\beta}{1-f\beta}\right)^{\alpha w n_i} - \left(\frac{1-\beta}{1-(1-f)\beta}\right)^{\alpha w n_i} \\
&\qquad + (1-\beta)^{\alpha w n_i}
\end{aligned} \tag{A3}$$

Derivation of Eq. 10

The terms on the right-hand side of Eq. 9 can be expressed as

$$\left(\frac{1-\beta}{1-f\beta}\right)^{\alpha w n_i} = \exp\left[\alpha w n_i \ln(1-\beta) - \alpha w n_i \ln(1-f\beta)\right]$$

$$\left(\frac{1-\beta}{1-(1-f)\beta}\right)^{\alpha w n_i} = \exp\left[\alpha w n_i \ln(1-\beta) - \alpha w n_i \ln\left(1-(1-f)\beta\right)\right]$$

$$(1-\beta)^{\alpha w n_i} = \exp\left[\alpha w n_i \ln(1-\beta)\right]$$

Hence, we have

$$\begin{aligned}\Pr(H_i \geq 1) &= \int_0^\infty \Pr(H_i \geq 1 | W_i = w) g(w) dw \\ &= \int_0^\infty \Big\{1 - \exp\left[\alpha w n_i \ln(1-\beta) - \alpha w n_i \ln(1-f\beta)\right] \\ &\quad - \exp\left[\alpha w n_i \ln(1-\beta) - \alpha w n_i \ln\left(1-(1-f)\beta\right)\right] \\ &\quad + \exp\left[\alpha w n_i \ln(1-\beta)\right]\Big\} \frac{1}{\Gamma(a)} b^a w^{a-1} \exp(-bw) dw\end{aligned} \tag{A4}$$

We can use an integral formula:

$$\int_0^\infty x^{g-1} \exp(-cx) dx = c^{-g}\Gamma(g) \tag{A5}$$

By applying the formula A5 for Eq. A4, we obtain Eq. 10.

Derivation of Eq. 15

Let us define $z - c = r$, for convenience. Then, we have

$$\begin{aligned}&\Pr(C_i = c | W_i = w) \\ &= \sum_{r=0}^\infty \binom{r+c}{c} p^c (1-p)^r \binom{-\alpha w n_i}{r+c} (-\beta)^{r+c} (1-\beta)^{\alpha w n_i}\end{aligned} \tag{A6}$$

We can use a combinatorial formula:

$$\binom{-\alpha w n_i}{r+c}\binom{r+c}{c} = \binom{-\alpha w n_i}{c}\binom{-\alpha w n_i - c}{r} \tag{A7}$$

Using Eqs. A2 and A7, we can express Eq. A6 as

$$\begin{aligned}&\Pr(C_i = c | W_i = w) \\ &= \binom{-\alpha w n_i}{c} (-p\beta)^c (1-\beta)^{\alpha w n_i} \cdot \sum_{r=0}^\infty \binom{-\alpha w n_i - c}{r} \left(-\beta(1-p)\right)^r \\ &= \binom{-\alpha w n_i}{c} \left(-\frac{\beta p}{1-\beta(1-p)}\right)^c \left(1 - \frac{\beta p}{1-\beta(1-p)}\right)^{\alpha w n_i}\end{aligned} \tag{A8}$$

Derivation of Eq. 27

Let x_i be the number of larvae in the ith infested fruit, m be the total number of infested fruit, i.e., $m = \sum_{i=1}^{k'} U'_i$. Then, the joint probability L of obtaining x_i is given by

$$L = \prod_{i=1}^m \frac{\alpha\beta^{x_i}}{x_i} = \prod_{i=1}^m \left[-\frac{\beta^{x_i}}{x_i \ln(1-\beta)}\right] \tag{A9}$$

By the differentiation of the logarithm of L about β, we have

$$\frac{\partial \ln(L)}{\partial \beta} = \sum_{i=1}^m \frac{x_i}{\beta} + \sum_{i=1}^m \frac{1}{(1-\beta)\ln(1-\beta)} \tag{A10}$$

Hence, we obtain:

$$\frac{1}{m}\sum_{i=1}^m x_i = -\frac{\hat{\beta}}{\left(1-\hat{\beta}\right)\ln\left(1-\hat{\beta}\right)} \tag{A11}$$

which yields Eq. 27. This equation coincides with the moment estimate of β.